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Charmless hadronic decays of B mesons to a pseudoscalar and a tensor meson

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Abstract. We study two-body charmless hadronic decays of B mesons to a pseudoscalar meson (P) and a tensor meson (T) in the frameworks of both flavor SU(3) symmetry and generalized factorization. Certain ways to test the validity of the generalized factorization are proposed, based on the flavor SU(3) analysis. We present a set of relations between a flavor SU(3) amplitude and the corresponding amplitude in the generalized factorization which bridge both approaches in $B \to PT$ decays. The branching ratios and CP asymmetries are calculated using the *full* effective Hamiltonian including all the *penguin* operators and the form factors obtained in the non-relativistic quark model of Isgur, Scora, Grinstein and Wise. We identify the decay modes in which the branching ratios and CP asymmetries are expected to be relatively large.

1 Introduction

The CLEO Collaboration has reported new experimental results on the branching ratios of a number of exclusive B meson decay modes where B decays into a pair of pseudoscalars (P), a vector (V) and a pseudoscalar meson, or a pair of vector mesons. Motivated by the new data, much work has been done to understand those exclusive hadronic B decays in the framework of the generalized factorization, QCD factorization, or flavor SU(3) symmetry. In the next few years B factories operating at SLAC and KEK will provide plenty of new experimental data on B decays. It is expected that an improved new bound will be put on the branching ratios for various decay modes, and that many decay modes with small branching ratios will be observed for the first time. Thus more information on rare decays of B mesons will be available soon.

There have been a few works [1–3] studying two-body hadronic B decays involving a tensor meson T ($J^P = 2^+$) in the final state using the non-relativistic quark model of Isgur, Scora, Grinstein and Wise (ISGW) [4] with the factorization ansatz. Most of them studied B decays involving a $b \to c$ transition, to which only the tree diagram contributes. In a recent work [3], the authors considered the Cabibbo-Kobayashi-Maskawa (CKM) suppressed hadronic B decays involving a $b \to u$ transition as well as a $b \to c$ transition. However, they included only the tree diagram contribution even in charmless B decays to PT and VT, such as $B \to \eta^{(\prime)} a_2$ and $B \to \phi f_2^{(\prime)}$. In most cases of the charmless $\Delta S = 0$ processes, the dominant contribution arises from the tree diagram and the contributions from the penguin diagrams are very small. But in some cases such as $B \to \eta^{(\prime)} a_2$ and $\eta^{(\prime)} f_2^{(\prime)}$, the penguin diagrams could provide sizable contributions.

Furthermore, in the charmless $|\Delta S| = 1$ decay processes, the penguin diagram contribution is enhanced by the CKM matrix elements $V_{tb}^* V_{ts}$ and becomes dominant. Experimentally several tensor mesons have been observed [5], such as the isovector $a_2(1320)$, the isoscalars $f_2(1270)$, $f_2'(1525)$, $f_2(2010)$, $f_2(2300)$, $f_2(2340)$, $\chi_{c2}(1P)$, $\chi_{b2}(1P)$ and $\chi_{c2}(2P)$, the isospinors $K_2^*(1430)$ and $D_2^*(2460)$. Experimental data on the branching ratios for B decays involving a pseudoscalar and a tensor meson in the final state provide only upper bounds, as follows [5]:

$$\begin{aligned}
\mathcal{B}(B^{+(0)} \to \pi^+ D_2^*(2460)^{0(-)}) &< 1.3(2.2) \times 10^{-3}, \\
\mathcal{B}(B^+ \to \pi^+ K_2^*(1430)^0) &< 6.8 \times 10^{-4}, \\
\mathcal{B}(B^+ \to \pi^+ f_2(1270)) &< 2.4 \times 10^{-4}, \\
\mathcal{B}(B^0 \to \pi^\pm a_2(1320)^\mp) &< 3.0 \times 10^{-4}. \qquad (1)
\end{aligned}$$

In this work, we analyze two-body charmless hadronic decays of B mesons to a pseudoscalar meson and a tensor meson in the frameworks of *both* flavor SU(3) symmetry and generalized factorization. Purely based on the flavor SU(3) symmetry, we first present a model-independent analysis in $B \to PT$ decays. Then we use the *full* effective Hamiltonian including all the penguin operators and the ISGW quark model to calculate the branching ratios for $B \to PT$ decays.

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Since we include both the tree and the penguin diagram contributions to decay processes, we are able to calculate the branching ratios for all the charmless $|\Delta S| = 1$ decays and the relevant CP asymmetries. In order to bridge the flavor SU(3) approach and the factorization approach, we present a set of relations between a flavor SU(3) amplitude and the corresponding amplitude in factorization in $B \to PT$ decays. Certain ways to test the validity of the generalized factorization are proposed by emphasizing the interplay between both approaches. We organize this work as follows. In Sect. 2 we discuss the notations for the SU(3) decomposition and the full effective Hamiltonian for B decays. In Sect. 3 we present a model-independent analysis of $B \to PT$ decays based on SU(3) symmetry. In Sect. 4 the two-body decays $B \to PT$ are analyzed in the framework of generalized factorization. The branching ratios and CP asymmetries are calculated using the form factors obtained in the ISGW quark model. Finally, in Sect. 5 our results are summarized.

2 Framework

In the flavor SU(3) approach, the decay amplitudes of two-body B decays are decomposed into linear combinations of the SU(3) amplitudes, which are the reduced matrix elements defined in [6]. In the SU(3) decomposition of the decay amplitudes of the $B \to PT$ processes, we choose the notations given in [6–8] as follows: We represent the decay amplitudes in terms of the basis of quark diagram contributions, T (tree), C (color-suppressed tree), P (QCD-penguin), S (additional penguin effect involving SU(3) singlet mesons), E (exchange), A (annihilation), and PA (penguin annihilation). The amplitudes E, A and PA may be neglected to a good approximation because of a suppression factor of $f_B/m_B \approx 5\%$. For later convenience we also denote the electroweak (EW) penguin effects explicitly by $P_{\rm EW}$ (color-favored EW penguin) and $P^C_{\rm EW}$ (color-suppressed EW penguin), even though in terms of quark diagrams the inclusion of these EW penguin effects only leads to the following replacement without introducing new SU(3) amplitudes: $T \to T + P^C_{\rm EW}$, $C \to C + P_{\rm EW}$, $P \to P - (1/3)P^C_{\rm EW}$, $S \to S - (1/3)P_{\rm EW}$. The phase convention used for the pseudoscalar and the tensor mesons is

$$
\begin{aligned}
\pi^+(a_2^+) &= u\bar{d}, \quad \pi^0(a_2^0) = -\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),\\
\pi^-(a_2^-) &= -\bar{u}d, \quad K^+(K_2^{*+}) = u\bar{s},\\
K^0(K_2^{*0}) &= d\bar{s}, \quad \bar{K}^0(\bar{K}_2^{*0}) = \bar{d}s,\\
K^-(K_2^{*-}) &= -\bar{u}s,\\
\eta &= -\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s}),\\
\eta' &= \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + 2s\bar{s}),\\
f_2 &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\phi_T + (s\bar{s})\sin\phi_T,\\
f_2' &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\phi_T - (s\bar{s})\cos\phi_T, \qquad (2)
\end{aligned}
$$

where the mixing angle ϕ_T is given by $\phi_T = \arctan(1/\sqrt{2}) - 28^0 \approx 7^0$ [1,9]. In the factorization scheme, we first consider the effective weak Hamiltonian. We then use the generalized factorization approximation to derive the hadronic matrix elements by saturating the vacuum state in all possible ways. The method includes a color octet non-factorizable contribution by treating $\xi \equiv 1/N_c$ (N_c denotes the effective number of color) as an adjustable parameter. The generalized factorization approximation has been quite successfully used in two-body D decays as well as $B \to D$ decays [10]. The effective weak Hamiltonian for hadronic $\Delta B = 1$ decays can be written as

$$
\begin{aligned}
H_{\rm eff} = \frac{4G_{\rm F}}{\sqrt{2}}\Bigg[&V_{ub}V_{uq}^*(c_1O_1^u + c_2O_2^u) \qquad (3)\\
&+V_{cb}V_{cq}^*(c_1O_1^c + c_2O_2^c) - V_{tb}V_{tq}^*\sum_{i=3}^{12}c_iO_i\Bigg] + \text{H.C.},
\end{aligned}
$$

where the O_i's are defined by

$$
\begin{aligned}
O_1^f &= (\bar{q}\gamma_\mu Lf)(\bar{f}\gamma^\mu Lb),\\
O_2^f &= (\bar{q}_\alpha\gamma_\mu Lf_\beta)(\bar{f}_\beta\gamma^\mu Lb_\alpha),\\
O_{3(5)} &= (\bar{q}\gamma_\mu Lb)(\Sigma\bar{q}'\gamma^\mu L(R)q'),\\
O_{4(6)} &= (\bar{q}_\alpha\gamma_\mu Lb_\beta)(\Sigma\bar{q}'_\beta\gamma^\mu L(R)q'_\alpha),\\
O_{7(9)} &= \frac{3}{2}(\bar{q}\gamma_\mu Lb)(\Sigma e_{q'}\bar{q}'\gamma^\mu R(L)q'),\\
O_{8(10)} &= \frac{3}{2}(\bar{q}_\alpha\gamma_\mu Lb_\beta)(\Sigma e_{q'}\bar{q}'_\beta\gamma^\mu R(L)q'_\alpha),\\
O_{11} &= \frac{g_s}{32\pi^2}m_b(\bar{q}\sigma^{\mu\nu}RT^ab)G^a_{\mu\nu},\\
O_{12} &= \frac{e}{32\pi^2}m_b(\bar{q}\sigma^{\mu\nu}Rb)F_{\mu\nu}. \qquad (4)
\end{aligned}
$$

Here the c_i's are the Wilson coefficients (WC's) evaluated at the renormalization scale μ. $L(R) = (1\mp\gamma_5)/2$, f can be a u or c quark, q can be a d or s quark, and q' is summed over u, d, s, and c quarks. α and β are the SU(3) color indices, and T^a ($a = 1, ..., 8$) are the SU(3) generators with the normalization $\text{Tr}(T^aT^b) = \delta^{ab}/2$. g_s and e are the strong and electric couplings, respectively. $G^a_{\mu\nu}$ and $F_{\mu\nu}$ denote the gluonic and photonic field strength tensors, respectively. O_1 and O_2 are the tree-level and QCD-corrected operators. $O_{3\text{-}6}$ are the gluon-induced strong penguin operators. $O_{7\text{-}10}$ are the EW penguin operators due to γ and Z exchange, and box diagrams at loop level. We shall take into account the chromomagnetic operator O_{11} but neglect the extremely small contribution from O_{12}. The dipole contribution is in general quite small, and is of the order of 10% for penguin dominated modes. For all the other modes it can be neglected [11].

We use the ISGW quark model to analyze two-body charmless decay processes $B \to PT$ in the framework of generalized factorization. We describe the parameterizations of the hadronic matrix elements in $B \to PT$ decays [4]:

$$\langle 0|A^\mu|P\rangle = \mathrm{i} f_P p_P^\mu, \tag{5}$$

$$\langle T|j^\mu|B\rangle = \mathrm{i}h(m_P^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}p_B^\alpha(p_B+p_T)_\rho(p_B-p_T)_\sigma + k(m_P^2)\epsilon^{*\mu\nu}(p_B)_\nu + \epsilon^*_{\alpha\beta}p_B^\alpha p_B^\beta \times[b_+(m_P^2)(p_B+p_T)^\mu + b_-(m_P^2)(p_B-p_T)^\mu], \tag{6}$$

where $j^\mu = V^\mu - A^\mu$. V^μ and A^μ denote a vector and an axial-vector current, respectively. f_P denotes the decay constant of the relevant pseudoscalar meson. $h(m_P^2)$, $k(m_P^2)$, $b_+(m_P^2)$, and $b_-(m_P^2)$ express the form factors for the $B \to T$ transition, $F^{B\to T}(m_P^2)$, which have been calculated at $q^2 = m_P^2$ ($q^\mu \equiv p_B^\mu - p_T^\mu$) in the ISGW quark model [4]. p_B and p_T denote the momentum of the B meson and the tensor meson, respectively. The polarization tensor $\epsilon^{\mu\nu}$ of the tensor meson T satisfies the following properties [12]:

$$\epsilon^{\mu\nu}(p_T,\lambda) = \epsilon^{\nu\mu}(p_T,\lambda), \tag{7}$$

$$p_\mu\epsilon^{\mu\nu}(p_T,\lambda) = p_\nu\epsilon^{\mu\nu}(p_T,\lambda) = 0, \tag{8}$$

$$\epsilon^\mu_\mu(p_T,\lambda) = 0, \tag{9}$$

where λ is the helicity index of the tensor meson. We note that due to the above properties of the polarization tensor, the matrix element $\langle 0|j^\mu|T\rangle$ vanishes:

$$\langle 0|j^\mu|T\rangle = p_\nu\epsilon^{\mu\nu}(p_T,\lambda) + p_T^\mu\epsilon^\nu_\nu(p_T,\lambda) = 0. \tag{10}$$

Thus, in the generalized factorization scheme, the decay amplitudes for $B \to PT$ can be considerably simplified, compared to those for other two-body charmless decays of B mesons such as $B \to PP$, PV, and VV: Any decay amplitude for $B \to PT$ is simply proportional to the decay constant f_P and a certain linear combination of the form factors $F^{B\to T}$, i.e., there is no such amplitude proportional to $f_T \times F^{B\to P}$.

3 Flavor SU(3) analysis of $B \to PT$ decays

We list the $B \to PT$ decay modes in terms of the SU(3) amplitudes. The coefficients of the SU(3) amplitudes in $B \to PT$ are listed in Tables 1 and 2 for strangeness-conserving ($\Delta S = 0$) and strangeness-changing ($|\Delta S| = 1$) processes, respectively. In the tables, the unprimed and the primed letters denote $\Delta S = 0$ and $|\Delta S| = 1$ processes, respectively. The subscript, P in T_P, C_P,... or T in T_T, C_T,... on each SU(3) amplitude is used to describe the particular case that the meson, which includes the spectator quark in the corresponding quark diagram, is the pseudoscalar P or the tensor T. Note that the coefficients of the SU(3) amplitudes with the subscript P, which would be proportional to $f_T \times F^{B\to P}$, are expressed in square brackets. As explained in Sect. 2, the contributions of the SU(3) amplitudes with the subscript P vanish in the framework of factorization, because those contributions contain the matrix element $\langle T|J_\mu^{\mathrm{weak}}|0\rangle$ which is zero; see (10).

Thus, it will be interesting to compare the results obtained in the SU(3) analysis with those obtained in the factorization scheme, as we shall see. We will present some ways to test the validity of both schemes in future experiments.

Among the $\Delta S = 0$ amplitudes, the tree diagram contribution is expected to be largest so that from Table 1 the decays $B^+ \to \pi^+ a_2^0$, $\pi^+ f_2$, and $B^0 \to \pi^+ a_2^-$ are expected to have the largest rates. Here we have noticed that in $B^+ \to \pi^+ f_2^{(\prime)}$ decays, $\cos\phi_T = 0.99$ and $\sin\phi_T = 0.13$, since the mixing angle $\phi_T \approx 7^0$. The amplitudes for the processes $B \to K K_2^*$ have only penguin diagram contributions, so they are expected to be small. In principle, the penguin contribution (combined with the smaller color-suppressed EW penguin) $p_T \equiv P_T - (1/3)P_{\mathrm{EW},T}$ can be measured in $B^{+(0)} \to \bar{K}^0 K_2^{*+(0)}$. The tree contribution (combined with the much smaller color-suppressed EW penguin) $t_T \equiv T_T + P^C_{\mathrm{EW},T}$ is measured by the combination $A(B^{+(0)} \to \bar{K}^0 K_2^{*+(0)}) - A(B^0 \to \pi^+ a_2^-)$. The amplitudes for $B^0 \to \pi^0 f_2'$, $\eta f_2'$, and $\eta' f_2'$ have color-suppressed tree contributions, $C_T(C_P)$, but are suppressed by $\sin\phi$ so that they are expected to be small. We shall see that these expectations based on the SU(3) approach are consistent with those calculated in the factorization approximation. However, there exist some cases in which the predictions based on both approaches are inconsistent. Note that in Table 1 the amplitudes for $B^0 \to \pi^- a_2^+$ and $B^{+(0)} \to K^{+(0)}\bar{K}_2^{*0}$ can be decomposed into linear combinations of the SU(3) amplitudes as follows:

$$A(B^0 \to \pi^- a_2^+) = -T_P - P_P - (2/3)P^C_{\mathrm{EW},P}, \tag{11}$$

$$A(B^+ \to K^+\bar{K}_2^{*0}) = A(B^0 \to K^0\bar{K}_2^{*0}) = P_P - (1/3)P^C_{\mathrm{EW},P}. \tag{12}$$

As previously explained, in factorization the rates for these processes vanish because all the SU(3) amplitudes have the subscript P.

Non-zero decay rates for these processes would arise from non-factorizable effects or final state interactions. Thus, in principle one can test the validity of the factorization ansatz by measuring the rates for these decays in future experiments. Therefore, the non-factorizable penguin contribution, if it exists (combined with the smaller color-suppressed EW penguin), $p_P \equiv P_P - (1/3)P_{\mathrm{EW},P}$ can be measured in $B^{+(0)} \to \bar{K}^{+(0)}\bar{K}_2^{*+(0)}$. Also, supposing that P_P is very small compared to T_P as usual, one can determine the magnitude of T_P by measuring the rate for $B^0 \to \pi^- a_2^+$. In the $|\Delta S| = 1$ decays, the (strong) penguin contribution P' is expected to dominate because of enhancement by the ratio of the CKM elements $|V_{tb}^* V_{ts}|/|V_{ub}^* V_{us}| \approx 50$. We note that the amplitudes for $B^+ \to K^0 a_2^+$ and $B^+ \to \pi^+ K_2^{*0}$ have only penguin contributions, respectively, as follows:

$$A(B^+ \to K^0 a_2^+) = P_T' - \frac{1}{3}P^{C\prime}_{\mathrm{EW},T}, \tag{13}$$

$$A(B^+ \to \pi^+ K_2^{*0}) = P_P' - \frac{1}{3}P^{C\prime}_{\mathrm{EW},P}. \tag{14}$$

Thus the penguin contribution (combined with the smaller color-suppressed EW penguin) $p_T' \equiv P_T' - (1/3)P^{C\prime}_{\mathrm{EW},T}$ is

Table 1. Coefficients of SU(3) amplitudes in $B \to PT$ ($\Delta S = 0$). The coefficients of the SU(3) amplitudes with the subscript P are expressed in square brackets. As explained in Sect. 2, the contributions of the SU(3) amplitudes with the subscript P vanish in the framework of factorization, because those contributions contain the matrix element $\langle T|J_\mu^{\rm weak}|0\rangle$, which is zero. Here c and s denote $\cos\phi_T$ and $\sin\phi_T$, respectively

$B \to PT$	factor	T_T $[T_P]$	C_T $[C_P]$	P_T $[P_P]$	S_T $[S_P]$	$P_{{\rm EW},T}$ $[P_{{\rm EW},P}]$	$P^C_{{\rm EW},T}$ $[P^C_{{\rm EW},P}]$
$B^+ \to \pi^+ a_2^0$	$-\frac{1}{\sqrt{2}}$	1	[1]	$1, [-1]$	0	[1]	$\frac{2}{3}$, $\left[\frac{1}{3}\right]$
$B^+ \to \pi^+ f_2$	$\frac{1}{\sqrt{2}}$	c	$[c]$	$c, [c]$	$[2c+\sqrt{2}s]$	$\left[\frac{1}{3}(c-\sqrt{2}s)\right]$	$2\frac{c}{3}$, $\left[-\frac{c}{3}\right]$
$B^+ \to \pi^+ f_2'$	$\frac{1}{\sqrt{2}}$	s	$[s]$	$s, [s]$	$[2s-\sqrt{2}c]$	$\left[\frac{1}{3}(s+\sqrt{2}c)\right]$	$2\frac{s}{3}$, $\left[-\frac{s}{3}\right]$
$B^+ \to \pi^0 a_2^+$	$-\frac{1}{\sqrt{2}}$	[1]	1	-1, [1]	0	1	$\frac{1}{3}$, $\left[\frac{2}{3}\right]$
$B^+ \to \eta a_2^+$	$-\frac{1}{\sqrt{3}}$	[1]	1	1, [1]	1	$\frac{2}{3}$	$-\frac{1}{3}$, $\left[\frac{2}{3}\right]$
$B^+ \to \eta' a_2^+$	$\frac{1}{\sqrt{6}}$	[1]	1	1, [1]	4	$-\frac{1}{3}$	$-\frac{1}{3}$, $\left[\frac{2}{3}\right]$
$B^+ \to K^+ \bar{K}_2^{*0}$	1	0	0	[1]	0	0	$\left[-\frac{1}{3}\right]$
$B^+ \to \bar{K}^0 K_2^{*+}$	1	0	0	1	0	0	$-\frac{1}{3}$
$B^0 \to \pi^+ a_2^-$	-1	1	0	1	0	0	$\frac{2}{3}$
$B^0 \to \pi^- a_2^+$	-1	[1]	0	[1]	0	0	$\left[\frac{2}{3}\right]$
$B^0 \to \pi^0 a_2^0$	$\frac{1}{2}$	0	-1, $[-1]$	1, [1]	0	-1, $[-1]$	$-\frac{1}{3}$, $\left[-\frac{1}{3}\right]$
$B^0 \to \pi^0 f_2$	$-\frac{1}{2}$	0	c, $[-c]$	$-c$, $[-c]$	$[-(2c+\sqrt{2}s)]$	c, $\left[-\frac{1}{3}(c-\sqrt{2}s)\right]$	$\frac{c}{3}$, $\left[\frac{c}{3}\right]$
$B^0 \to \pi^0 f_2'$	$-\frac{1}{2}$	0	s, $[-s]$	$-s$, $[-s]$	$[-(2s-\sqrt{2}c)]$	s, $\left[-\frac{1}{3}(s+\sqrt{2}c)\right]$	$\frac{s}{3}$, $\left[\frac{s}{3}\right]$
$B^0 \to \eta a_2^0$	$\frac{1}{\sqrt{6}}$	0	-1, [1]	-1, $[-1]$	-1	$-\frac{2}{3}$, [1]	$\frac{1}{3}$, $\left[\frac{1}{3}\right]$
$B^0 \to \eta f_2$	$-\frac{1}{\sqrt{6}}$	0	c, $[c]$	c, $[c]$	c, $[2c+\sqrt{2}s]$	$2\frac{c}{3}$, $\left[\frac{1}{3}(c-\sqrt{2}s)\right]$	$-\frac{c}{3}$, $\left[-\frac{c}{3}\right]$
$B^0 \to \eta f_2'$	$-\frac{1}{\sqrt{6}}$	0	s, $[s]$	s, $[s]$	s, $[2s-\sqrt{2}c]$	$2\frac{s}{3}$, $\left[\frac{1}{3}(s+\sqrt{2}c)\right]$	$-\frac{s}{3}$, $\left[-\frac{s}{3}\right]$
$B^0 \to \eta' a_2^0$	$-\frac{1}{2\sqrt{3}}$	0	-1, [1]	-1, $[-1]$	-4	$\frac{1}{3}$, [1]	$\frac{1}{3}$, $\left[\frac{1}{3}\right]$
$B^0 \to \eta' f_2$	$\frac{1}{2\sqrt{3}}$	0	c, $[c]$	c, $[c]$	$4c$, $[2c+\sqrt{2}s]$	$-\frac{c}{3}$, $\left[\frac{1}{3}(c-\sqrt{2}s)\right]$	$-\frac{c}{3}$, $\left[-\frac{c}{3}\right]$
$B^0 \to \eta' f_2'$	$\frac{1}{2\sqrt{3}}$	0	s, $[s]$	s, $[s]$	$4s$, $[2s-\sqrt{2}c]$	$-\frac{s}{3}$, $\left[\frac{1}{3}(s+\sqrt{2}c)\right]$	$-\frac{s}{3}$, $\left[-\frac{s}{3}\right]$
$B^0 \to K^0 \bar{K}_2^{*0}$	1	0	0	[1]	0	0	$\left[-\frac{1}{3}\right]$
$B^0 \to \bar{K}^0 K_2^{*0}$	1	0	0	1	0	0	$-\frac{1}{3}$

measured in $B^+ \to K^0 a_2^+$. Similarly, $p_P' \equiv P_P' - (1/3) P_{{\rm EW},P}^{C\prime}$ is determined in $B^+ \to \pi^+ K_2^{*0}$. (In fact, $p_P' = 0$ in factorization.) By comparing the branching ratios for these two modes measured in experiment, one can determine which contribution (i.e., p_T' or p_P') is larger. The (additional penguin) SU(3) singlet amplitude S' is expected to be very small because of the Okubo–Zweig–Iizuka (OZI) suppression, but the SU(3) singlet amplitude S' for the decays involving the pseudoscalar mesons η and η' is expected not to be very small, since the flavor-singlet couplings of the η and η' can be affected by the axial anomaly [13]. Thus, from Table 2, one can expect that the processes $B^{+(0)} \to \eta' K_2^{*+(0)}$ have large branching ratios compared to other $|\Delta S| = 1$ decays, since they have both the penguin contributions P' and S' (and the smaller EW penguin contributions $P_{\rm EW}'$ and $P_{\rm EW}^{C\prime}$) and these contributions interfere constructively like $2P_T' + P_P' + 4S_T'$. In contrast, the processes $B^{+(0)} \to \eta K_2^{*+(0)}$ have the penguin contributions P' and S', but these interfere destructively like $-P_T' + P_P' + S_T'$. As in $\Delta S = 0$ decays, there are certain processes whose amplitudes can be expressed by the SU(3) amplitudes, but which are expected to vanish in factorization: For instance, $A(B^+ \to \pi^+ K_2^{*0})$ is given by (14) and $A(B^0 \to \pi^- K_2^{*+}) = -\left(T_P' + P_P' + (2/3) P_{{\rm EW},P}^{C\prime}\right)$. Thus, in principle measurement of the rates for these decays can be used to test the factorization ansatz. We also note that the decay amplitudes for modes $B^+ \to \pi^0 K_2^{*+}$ and $B^0 \to \pi^0 K_2^{*0}$ can be respectively written

$$
\begin{aligned}
&A(B^+ \to \pi^0 K_2^{*+}) \\
&\quad = -\frac{1}{\sqrt{2}} \left(T_P' + C_T' + P_P' + P_{{\rm EW},T}' + \frac{2}{3} P_{{\rm EW},P}^{C\prime} \right), \\
&A(B^0 \to \pi^0 K_2^{*0}) \\
&\quad = -\frac{1}{\sqrt{2}} \left(C_T' - P_P' + P_{{\rm EW},T}' + \frac{1}{3} P_{{\rm EW},P}^{C\prime} \right).
\end{aligned} \tag{15, 16}
$$

Since in factorization only the amplitudes having the subscript T do not vanish, we shall see that $\mathcal{B}(B^+ \to \pi^0 K_2^{*+}) = \mathcal{B}(B^0 \to \pi^0 K_2^{*0})$ in the factorization scheme, where $\mathcal{B}$ denotes the branching ratio. Thus, if P_P' or T_P' is (not zero and) not very suppressed compared to C_T', then there would be a sizable discrepancy in the relation $\mathcal{B}(B^+ \to \pi^0 K_2^{*+}) = \mathcal{B}(B^0 \to \pi^0 K_2^{*0})$, and in principle it can be tested in experiment. From Tables 1 and 2, we find some useful relations among the decay amplitudes. The equivalence relations are

$$
\begin{aligned}
A(B^+ \to K^+ \bar{K}_2^{*0}) &= A(B^0 \to K^0 \bar{K}_2^{*0}), \\
A(B^+ \to \bar{K}^0 K_2^{*+}) &= A(B^0 \to \bar{K}^0 K_2^{*0}).
\end{aligned} \tag{17}
$$

The quadrangle relations are for the $\Delta S = 0$ processes:

$$
\sqrt{2} A(B^+ \to \eta' a_2^+) + A(B^+ \to \eta a_2^+)
$$

Table 2. Coefficients of SU(3) amplitudes in $B \to PT$ ($|\Delta S| = 1$)

$B \to PT$	factor	T'_T $[T'_P]$	C'_T $[C'_P]$	P'_T $[P'_P]$	S'_T $[S'_P]$	$P'_{\mathrm{EW},T}$ $[P'_{\mathrm{EW},P}]$	$P^{C\prime}_{\mathrm{EW},T}$ $[P^{C\prime}_{\mathrm{EW},P}]$
$B^+ \to K^+ a_2^0$	$-\frac{1}{\sqrt{2}}$	1	[1]	1	0	[1]	$\frac{2}{3}$
$B^+ \to K^+ f_2$	$\frac{1}{\sqrt{2}}$	c	$[c]$	c, $[\sqrt{2}s]$	$[2c+\sqrt{2}s]$	$\left[\frac{1}{3}(c-\sqrt{2}s)\right]$	$2\frac{c}{3}$, $\left[-\sqrt{2}\frac{s}{3}\right]$
$B^+ \to K^+ f'_2$	$\frac{1}{\sqrt{2}}$	s	$[s]$	s, $[-\sqrt{2}c]$	$[2s-\sqrt{2}c]$	$\left[\frac{1}{3}(s+\sqrt{2}c)\right]$	$2\frac{s}{3}$, $\left[\sqrt{2}\frac{c}{3}\right]$
$B^+ \to K^0 a_2^+$	1	0	0	1	0	0	$-\frac{1}{3}$
$B^+ \to \pi^+ K_2^{*0}$	1	0	0	[1]	0	0	$\left[-\frac{1}{3}\right]$
$B^+ \to \pi^0 K_2^{*+}$	$-\frac{1}{\sqrt{2}}$	[1]	1	[1]	0	1	$\left[\frac{2}{3}\right]$
$B^+ \to \eta K_2^{*+}$	$-\frac{1}{\sqrt{3}}$	[1]	1	-1, [1]	1	$\frac{2}{3}$	$\frac{1}{3}$, $\left[\frac{2}{3}\right]$
$B^+ \to \eta' K_2^{*+}$	$\frac{1}{\sqrt{6}}$	[1]	1	2, [1]	4	$-\frac{1}{3}$	$-\frac{2}{3}$, $\left[\frac{2}{3}\right]$
$B^0 \to K^+ a_2^-$	-1	1	0	1	0	0	$\frac{2}{3}$
$B^0 \to K^0 a_2^0$	$-\frac{1}{\sqrt{2}}$	0	[1]	-1	0	[1]	$\frac{1}{3}$
$B^0 \to K^0 f_2$	$\frac{1}{\sqrt{2}}$	0	$[c]$	c, $[\sqrt{2}s]$	$[2c+\sqrt{2}s]$	$\left[\frac{1}{3}(c-\sqrt{2}s)\right]$	$-\frac{c}{3}$, $\left[-\sqrt{2}\frac{s}{3}\right]$
$B^0 \to K^0 f'_2$	$\frac{1}{\sqrt{2}}$	0	$[s]$	s, $[-\sqrt{2}c]$	$[2s-\sqrt{2}c]$	$\left[\frac{1}{3}(s+\sqrt{2}c)\right]$	$-\frac{s}{3}$, $\left[\sqrt{2}\frac{c}{3}\right]$
$B^0 \to \pi^- K_2^{*+}$	-1	[1]	0	[1]	0	0	$\left[\frac{2}{3}\right]$
$B^0 \to \pi^0 K_2^{*0}$	$-\frac{1}{\sqrt{2}}$	0	1	$[-1]$	0	1	$\left[\frac{1}{3}\right]$
$B^0 \to \eta K_2^{*0}$	$-\frac{1}{\sqrt{3}}$	0	1	-1, [1]	1	$\frac{2}{3}$	$\frac{1}{3}$, $\left[-\frac{1}{3}\right]$
$B^0 \to \eta' K_2^{*0}$	$\frac{1}{\sqrt{6}}$	0	1	2, [1]	4	$-\frac{1}{3}$	$-\frac{2}{3}$, $\left[-\frac{1}{3}\right]$

$$
\begin{aligned}
&= 2A(B^0 \to \eta' a_2^0) + \sqrt{2}A(B^0 \to \eta a_2^0), \\
&\frac{1}{c}[A(B^+ \to \pi^+ f_2) - \sqrt{2}A(B^0 \to \pi^0 f_2)] \\
&= \frac{1}{s}[A(B^+ \to \pi^+ f'_2) - \sqrt{2}A(B^0 \to \pi^0 f'_2)] \\
&= \frac{1}{c}[\sqrt{2}A(B^0 \to \eta' f_2) + A(B^0 \to \eta f_2)] \\
&= \frac{1}{s}[\sqrt{2}A(B^0 \to \eta' f'_2) + A(B^0 \to \eta f'_2)], \qquad (18)
\end{aligned}
$$

and for the $|\Delta S| = 1$ processes:

$$
\begin{aligned}
&\sqrt{2}A(B^+ \to K^+ a_2^0) + A(B^+ \to K^0 a_2^+) \\
&= A(B^0 \to K^+ a_2^-) + \sqrt{2}A(B^0 \to K^0 a_2^0), \\
&\frac{1}{c}[A(B^+ \to K^+ f_2) - A(B^0 \to K^0 f_2)] \\
&= \frac{1}{s}[A(B^+ \to K^+ f'_2) - A(B^0 \to K^0 f'_2)], \\
&A(B^+ \to \pi^+ K_2^{*0}) + \sqrt{2}A(B^+ \to \pi^0 K_2^{*+}) \\
&= A(B^0 \to \pi^- K_2^{*+}) + \sqrt{2}A(B^0 \to \pi^0 K_2^{*0}), \\
&A(B^+ \to \eta K_2^{*+}) + \sqrt{2}A(B^+ \to \eta' K_2^{*+}) \\
&= A(B^0 \to \eta K_2^{*0}) + \sqrt{2}A(B^0 \to \eta' K_2^{*0}), \qquad (19)
\end{aligned}
$$

where $c \equiv \cos\phi_T$ and $s \equiv \sin\phi_T$. Note that the above relations are derived purely based on flavor SU(3) symmetry. In the factorization scheme (neglecting the SU(3) amplitudes with the subscript P) we would have in addition the approximate relations as follows[1]. The following factorization relation would hold:

$$
\sqrt{2}A(B^+ \to \pi^+ a_2^0) \approx A(B^0 \to \pi^+ a_2^-). \qquad (20)
$$

[1] Considering SU(3) breaking effects, we use the symbol $\approx$ in the following relations instead of the equivalence symbol $=$

The quadrangle relations given in (18) and (19) would be divided into the following factorization relations: for the $\Delta S = 0$ processes,

$$
\begin{aligned}
A(B^+ \to \eta a_2^+) &\approx \sqrt{2}A(B^0 \to \eta a_2^0) \\
&\approx -\sqrt{2}A(B^+ \to \eta' a_2^+) \\
&\approx -2A(B^0 \to \eta' a_2^0), \\
\frac{1}{c}A(B^{+(0)} \to \pi^{+(0)} f_2) &\approx \frac{1}{s}A(B^{+(0)} \to \pi^{+(0)} f'_2), \\
\frac{1}{c}A(B^0 \to \eta f_2) &\approx \frac{1}{s}A(B^0 \to \eta f'_2) \\
&\approx -\frac{1}{c}\sqrt{2}A(B^0 \to \eta' f_2) \\
&\approx -\frac{1}{s}\sqrt{2}A(B^0 \to \eta' f'_2), \qquad (21)
\end{aligned}
$$

and for the $|\Delta S| = 1$ processes:

$$
\begin{aligned}
\sqrt{2}A(B^+ \to K^+ a_2^0) &\approx A(B^0 \to K^+ a_2^-), \\
A(B^+ \to K^0 a_2^+) &\approx \sqrt{2}A(B^0 \to K^0 a_2^0), \\
\frac{1}{c}A(B^+ \to K^+ f_2) &\approx \frac{1}{s}A(B^+ \to K^+ f'_2), \\
\frac{1}{c}A(B^0 \to K^0 f_2) &\approx \frac{1}{s}A(B^0 \to K^0 f'_2), \\
A(B^+ \to \pi^0 K_2^{*+}) &\approx A(B^0 \to \pi^0 K_2^{*0}), \\
A(B^+ \to \eta K_2^{*+}) &\approx A(B^0 \to \eta K_2^{*0}), \\
A(B^+ \to \eta' K_2^{*+}) &\approx A(B^0 \to \eta' K_2^{*0}). \qquad (22)
\end{aligned}
$$

Therefore, in principle the above relations given in (20), (21) and (22) provide an interesting way to test the factorization scheme by measuring and comparing magnitudes of the decay amplitudes involved in the relations. In consideration of SU(3) breaking effects, the relation in (20) is best to use, because in fact the relation arises from isospin symmetry assuming $C_P = P_P = P_{\rm EW,P} = P^C_{\rm EW,P} = 0$. (However, if C_P is negligibly small (though not zero) compared to T_T, (20) will approximately hold.)

4 Analysis of $B \to PT$ decays using the Isgur-Scora-Grinsteing-Wise model

Now, we present expressions for SU(3) amplitudes involved in $B \to PT$ decays as calculated in the factorization scheme as follows [14] (note that all the SU(3) amplitudes with the subscript P vanish because those are proportional to the matrix element $\langle T|j^\mu|0\rangle$):

$$\begin{aligned}
T_T^{(\prime)} &= {\rm i}\frac{G_{\rm F}}{\sqrt{2}} V^*_{ub}V_{ud(s)}(f_P \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to T}(m_P^2))a_1, \\
C_T^{(\prime)} &= {\rm i}\frac{G_{\rm F}}{\sqrt{2}} V^*_{ub}V_{ud(s)}(f_P \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to T}(m_P^2))a_2, \\
S_T^{(\prime)} &= -{\rm i}\frac{G_{\rm F}}{\sqrt{2}} V^*_{tb}V_{td(s)}(f_P \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to T}(m_P^2)) \\
&\quad \times (a_3 - a_5), \\
P_T^{(\prime)} &= -{\rm i}\frac{G_{\rm F}}{\sqrt{2}} V^*_{tb}V_{td(s)}(f_P \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to T}(m_P^2)) \\
&\quad \times (a_4 - 2a_6 X_{qq'}), \\
P_{{\rm EW},T}^{(\prime)} &= -{\rm i}\frac{G_{\rm F}}{\sqrt{2}} V^*_{tb}V_{td(s)}(f_P \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to T}(m_P^2)) \\
&\quad \times \frac{3}{2}(a_7 - a_9), \\
P_{{\rm EW},T}^{C(\prime)} &= -{\rm i}\frac{G_{\rm F}}{\sqrt{2}} V^*_{tb}V_{td(s)}(f_P \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to T}(m_P^2)) \\
&\quad \times \frac{3}{2}(a_{10} - 2a_8 X_{qq'}),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
F^{B\to T}(m_P^2) &= k(m_P^2) + (m_B^2 - m_T^2)b_+(m_P^2) \\
&\quad + m_P^2 b_-(m_P^2),
\end{aligned} \tag{24}$$

$$X_{qq'} = \frac{m_P^2}{(m_b + m_{q'})(m_q + m_{q'})}. \tag{25}$$

Here the effective coefficients a_i are defined by $a_i = c_i^{\rm eff} + \xi c_{i+1}^{\rm eff}$ (i is odd) and $a_i = c_i^{\rm eff} + \xi c_{i-1}^{\rm eff}$ (i is even) with the effective WC's $c_i^{\rm eff}$ at the scale m_b [11,15], and by treating $\xi \equiv 1/N_c$ (N_c denotes the effective number of color) as an adjustable parameter. The last term with b_- in (24) gives a negligible contribution to the decay amplitude due to the small mass factor. With Tables 1, 2 and the above relations (23), one can easily write down in the factorization scheme the amplitude of any $B \to PT$ mode shown in the tables. For example, from Table 1 and the relations (23), the amplitude of the process $B^+ \to \pi^+ a_2^0$ can be written[2]

$$\begin{aligned}
A(B^+ \to \pi^+ a_2^0) &= -\frac{1}{\sqrt{2}} \\
&\quad \times \left(T_T + C_P + P_T - P_P + P_{\rm EW,P} + \frac{2}{3}P^C_{{\rm EW},T} \right.\\
&\quad \left. + \frac{1}{3}P^C_{{\rm EW},P}\right) \\
&= {\rm i}\frac{G_{\rm F}}{2} f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^0}(m_\pi^2) \\
&\quad \times \{V^*_{ub}V_{ud}a_1 - V^*_{tb}V_{td}[a_4 + a_{10} - 2(a_6 + a_8)X_{du}]\}.
\end{aligned} \tag{26}$$

[2] In the factorization scheme, we use the usual phase convention for the pseudoscalar and the tensor mesons as follows: $\pi^0(a_2^0) = (1/\sqrt{2})(u\bar{u} - d\bar{d})$, $\pi^-(a_2^-) = \bar{u}d$, $K^-(K_2^{*-}) = \bar{u}s$

Here we have used the fact that C_P, P_P, $P_{{\rm EW},P}$, and $P^C_{{\rm EW},P}$ with the subscript P all vanish. In the appendix, expressions for all the amplitudes of $B \to PT$ decays are presented as calculated in the factorization scheme.

To calculate the unpolarized decay rates for $B \to PT$, we sum over polarizations of the tensor meson T using the following formula [2]:

$$\sum_\lambda \epsilon_{\alpha\beta}(p_T,\lambda)\epsilon^*_{\mu\nu}(p_T,\lambda) = \frac{1}{2}(\theta_{\alpha\mu}\theta_{\beta\nu} + \theta_{\beta\mu}\theta_{\alpha\nu}) - \frac{1}{3}\theta_{\alpha\beta}\theta_{\mu\nu}, \tag{27}$$

where $\theta_{\alpha\beta} = -g_{\alpha\beta} + (p_T)_\alpha(p_T)_\beta/m_T^2$. Then, the decay rate for $B \to PT$ is given by

$$\Gamma(B \to PT) = \frac{|\boldsymbol{p}_P|^5}{12\pi m_T^2}\left(\frac{m_B}{m_T}\right)^2 \left|\frac{A(B \to PT)}{\epsilon^*_{\mu\nu}p^\mu_B p^\nu_B}\right|^2, \tag{28}$$

where $|\boldsymbol{p}_P|$ is the magnitude of the three-momentum of the final state particle P or T ($|\boldsymbol{p}_P| = |\boldsymbol{p}_T|$) in the rest frame of the B meson. The CP asymmetry, $\mathcal{A}_{CP}$, is defined by

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(b \to f) - \mathcal{B}(\bar{b} \to \bar{f})}{\mathcal{B}(b \to f) + \mathcal{B}(\bar{b} \to \bar{f})}, \tag{29}$$

where b and f denote b quark and a generic final state, respectively.

We calculate the branching ratios and the CP asymmetries for the $B \to PT$ decay modes for various input parameter values. The predictions are sensitive to several input parameters such as the form factors, the strange quark mass, the parameter $\xi \equiv 1/N_c$, the CKM matrix elements and, in particular, the weak phase γ. In a recent work [11] on charmless B decays to two light mesons such as PP and VP, it has been shown that the favored values of the input parameters are

$$\xi \approx 0.45, \quad m_s(m_b) \approx 85\,{\rm MeV}, \quad \gamma \approx 110^\circ,$$
$$V_{cb} = 0.040, \quad \text{and} \quad |V_{ub}/V_{cb}| = 0.087,$$

in order to get the best fit to the recent experimental data from the CLEO collaboration. For our numerical calculations, we use the following values of the decay constants (in MeV units) [10,15,16]:

$$f_\pi = 132, \quad f_\eta = 131, \quad f_{\eta'} = 118, \quad f_K = 162.$$

We use the values of the form factors for the $B \to T$ transition calculated in the ISGW model [4]. The strange quark mass m_s is in considerable doubt: i.e., QCD sum rules give $m_s(1\,{\rm GeV}) = (175 \pm 25)\,{\rm MeV}$ and lattice gauge theory gives $m_s(2\,{\rm GeV}) = (100 \pm 20 \pm 10)\,{\rm MeV}$ in the

Table 3. The branching ratios for $B \to PT$ decay modes with $\Delta S = 0$. The second and the third columns correspond to the cases of sets of the parameters: $\{\xi = 0.1,\ m_s = 85\,\mathrm{MeV},\ \gamma = 110°\}$ and $\{\xi = 0.1,\ m_s = 100\,\mathrm{MeV},\ \gamma = 65°\}$, respectively. Similarly, the fourth and the fifth columns corresponds to the cases: $\{\xi = 0.3,\ m_s = 85\,\mathrm{MeV},\ \gamma = 110°\}$ and $\{\xi = 0.3,\ m_s = 100\,\mathrm{MeV},\ \gamma = 65°\}$, respectively. The sixth and the seventh columns correspond to the cases: $\{\xi = 0.5,\ m_s = 85\,\mathrm{MeV},\ \gamma = 110°\}$ and $\{\xi = 0.5,\ m_s = 100\,\mathrm{MeV},\ \gamma = 65°\}$, respectively

Decay mode	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$
$B^+ \to \pi^+ a_2^0$	45.41	44.82	40.32	39.82	35.54	35.11
$B^+ \to \pi^+ f_2$	49.31	48.67	43.79	43.24	38.59	38.13
$B^+ \to \pi^+ f_2'$	0.46	0.46	0.41	0.40	0.36	0.36
$B^+ \to \pi^0 a_2^+$	1.78	1.52	0.029	0.048	2.05	2.38
$B^+ \to \eta a_2^+$	5.81	6.02	5.20	3.94	7.09	4.48
$B^+ \to \eta' a_2^+$	27.19	22.97	23.02	17.93	20.33	14.45
$B^+ \to \bar{K}^0 K_2^{*+}$	0.025	0.013	0.032	0.019	0.041	0.026
$B^0 \to \pi^+ a_2^-$	85.91	84.80	76.29	75.34	67.23	66.44
$B^0 \to \pi^0 a_2^0$	0.84	0.72	0.014	0.023	0.97	1.12
$B^0 \to \pi^0 f_2$	0.92	0.78	0.015	0.025	1.05	1.22
$B^0 \to \pi^0 f_2'$	0.009	0.007	0.0001	0.0001	0.010	0.011
$B^0 \to \eta a_2^0$	2.75	2.85	2.46	1.86	3.36	2.12
$B^0 \to \eta f_2$	2.99	3.09	2.67	2.02	3.65	2.30
$B^0 \to \eta f_2'$	0.03	0.03	0.025	0.019	0.024	0.021
$B^0 \to \eta' a_2^0$	12.86	10.87	10.89	8.48	9.62	6.83
$B^0 \to \eta' f_2$	14.00	10.87	11.85	9.23	10.47	7.44
$B^0 \to \eta' f_2'$	0.13	0.11	0.11	0.085	0.096	0.068
$B^0 \to \bar{K}^0 K_2^{*0}$	0.023	0.012	0.030	0.017	0.038	0.024

quenched lattice calculation [17]. In this analysis we use two representative values of $m_s = 100\,\mathrm{MeV}$ and $m_s = 85\,\mathrm{MeV}$ at m_b scale. Current best estimates for CKM matrix elements are $V_{cb} = 0.0381 \pm 0.0021$ and $|V_{ub}/V_{cb}| = 0.085 \pm 0.019$ [18]. We use $V_{cb} = 0.040$ and $|V_{ub}/V_{cb}| = 0.087$. It is known that there exists a discrepancy in the values of γ extracted from CKM fitting in the ρ–η plane [19] and from the χ^2 analysis of non-leptonic decays of the B mesons [20,21]. The value of γ obtained from unitarity triangle fitting is in the range of $60° \sim 80°$. But in the analysis of non-leptonic B decay, the possibility of a larger γ has been discussed by Deshpande et al. [20] and He et al. [21]. The obtained value of γ is $\gamma = 90° \sim 140°$. In our calculations we use the two representative values of $\gamma = 110°$ and $\gamma = 65°$. In Tables 3–6, we show the branching ratios and the CP asymmetries for $B \to PT$ decays with either $\Delta S = 0$ or $|\Delta S| = 1$. In the tables the second and the third columns correspond to the sets of input parameters

$$\{\xi = 0.1, m_s = 85\,\mathrm{MeV}, \gamma = 110^0\}$$

and

$$\{\xi = 0.1, m_s = 100\,\mathrm{MeV}, \gamma = 65^0\},$$

respectively. Similarly, the fourth and the fifth columns correspond to the cases

$$\{\xi = 0.3, m_s = 85\,\mathrm{MeV}, \gamma = 110°\}$$

and

$$\{\xi = 0.3, m_s = 100\,\mathrm{MeV}, \gamma = 65°\},$$

respectively. The sixth and the seventh columns correspond to the cases

$$\{\xi = 0.5, m_s = 85\,\mathrm{MeV}, \gamma = 110°\}$$

and

$$\{\xi = 0.5, m_s = 100\,\mathrm{MeV}, \gamma = 65°\},$$

respectively. Here $\xi \equiv 1/N_c = 0.3$ corresponds to the case of naive factorization ($N_c = 3$). It is known that in $B \to D$ decays the generalized factorization has been successfully used with the favored value of $\xi \approx 0.5$ [22]. Also, as mentioned above, a recent analysis of charmless B decays to two light mesons such as PP and VP [11] shows that $\xi \approx 0.45$ is favored with certain values of the other parameters for the best fit to the recent CLEO data.

The branching ratios and the CP asymmetries for $B \to PT$ decay modes with $\Delta S = 0$ are shown in Table 3 and 4. Among the $\Delta S = 0$ modes, the decay modes $B^+ \to \pi^+ a_2^0$, $B^+ \to \pi^+ f_2$, and $B^0 \to \pi^+ a_2^-$ have the relatively large branching ratios of a few times 10^{-7}. This prediction is consistent with that based on flavor SU(3) symmetry. We see that in the factorization scheme the following equality between the branching ratios holds for any set of parameters given above: $2\mathcal{B}(B^+ \to \pi^+ a_2^0) \approx \mathcal{B}(B^0 \to \pi^+ a_2^-)$,

Table 4. The CP asymmetries for $B \to PT$ decay modes with $\Delta S = 0$. The definitions for the columns are the same as those in Table 3

Decay mode	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$
$B^+ \to \pi^+ a_2^0$	0.016	0.016	0.015	0.015	0.015	0.015
$B^+ \to \pi^+ f_2$	0.016	0.016	0.015	0.015	0.015	0.015
$B^+ \to \pi^+ f_2'$	0.016	0.016	0.015	0.015	0.015	0.015
$B^+ \to \pi^0 a_2^+$	0.14	0.15	-0.89	-0.52	-0.13	-0.10
$B^+ \to \eta a_2^+$	0.59	0.55	-0.068	-0.087	-0.46	-0.71
$B^+ \to \eta' a_2^+$	0.17	0.20	-0.021	-0.026	-0.22	-0.29
$B^+ \to \bar{K}^0 K_2^{*+}$	0	0	0	0	0	0
$B^0 \to \pi^+ a_2^-$	0.016	0.015	0.015	0.015	0.015	0.015
$B^0 \to \pi^0 a_2^0$	0.14	0.15	-0.89	-0.52	-0.13	-0.10
$B^0 \to \pi^0 f_2$	0.14	0.15	-0.89	-0.52	-0.13	-0.10
$B^0 \to \pi^0 f_2'$	0.14	0.14	-0.89	-0.52	-0.13	-0.10
$B^0 \to \eta a_2^0$	0.59	0.55	-0.068	-0.087	-0.46	-0.71
$B^0 \to \eta f_2$	0.59	0.59	-0.068	-0.087	-0.46	-0.71
$B^0 \to \eta f_2'$	0.59	0.55	-0.068	-0.087	-0.46	-0.71
$B^0 \to \eta' a_2^0$	0.17	0.20	-0.021	-0.026	0.22	-0.29
$B^0 \to \eta' f_2$	0.17	0.20	-0.021	-0.026	-0.22	-0.29
$B^0 \to \eta' f_2'$	0.17	0.20	-0.021	-0.026	-0.22	-0.29
$B^0 \to \bar{K}^0 K_2^{*0}$	0	0	0	0	0	0

as discussed in (20). (A little deviation from the exact equality arises from breaking of isospin symmetry.) We also see from Table 3 that $\mathcal{B}(B^+ \to \pi^0 a_2^+)$ is much smaller than $\mathcal{B}(B^+ \to \pi^+ a_2^0)$ by an order of magnitude or even three orders of magnitude depending on the values of the input parameters, because in factorization the dominant contribution to the former mode arises from the color-suppressed tree diagram (C_T), while the dominant one to the latter mode arises from the color-favored tree diagram (T_T). Note that in flavor SU(3) symmetry the amplitude for $B^+ \to \pi^+ a_2^0$ has the color-favored tree contribution T_P constructive to the color-suppressed tree contribution C_T (in addition to small contributions from the penguin diagrams). (Also recall that the magnitude of T_P can be possibly measured by (11).) In case that T_P is not small compared to T_T, $\mathcal{B}(B^+ \to \pi^0 a_2^+)$ can be comparable to $\mathcal{B}(B^+ \to \pi^+ a_2^0)$. Therefore, measurement of the modes $B \to \pi a_2$ in future experiments will provide important information on the above discussion. Some $\Delta S = 0$ processes such as $B^+ \to \eta' a_2^+$, $B^0 \to \eta' a_2^0$, and $B^0 \to \eta' f_2$ have branching ratios of order of 10^{-7}. The branching ratios of the other processes are order of 10^{-8} or less. The CP asymmetry for $B^+ \to \eta' a_2^+$ is relatively large (about 20% or larger) with a branching ratio of order of 10^{-7} for $\xi = 0.5$ and 0.1. The CP asymmetry for $B^+ \to \eta a_2^+$, $B^0 \to \eta a_2^0$, ηf_2 can be as large as 71% for $\xi = 0.5$, with the branching ratios of $O(10^{-8})$.

Tables 5 shows the branching ratios for $|\Delta S| = 1$ decay processes. In $|\Delta S| = 1$ decays, the relevant penguin diagrams give a dominant contribution to the decay rates. We see that the branching ratios for $|\Delta S| = 1$ decays are in the range between $O(10^{-7})$ and $O(10^{-10})$, similar to those for $\Delta S = 0$ decays. The modes $B^{+(0)} \to \eta' K_2^{*+(0)}$ have relatively larger branching ratios of $O(10^{-7})$. In contrast, the modes $B^{+(0)} \to \eta K_2^{*+(0)}$ have the very small branching ratios of $O(10^{-9})$ to $O(10^{-10})$. Based on flavor SU(3) symmetry, this fact has been expected by the observation that the penguin contributions P' and S' interfere constructively for $B^{+(0)} \to \eta' K_2^{*+(0)}$, but destructively for $B^{+(0)} \to \eta K_2^{*+(0)}$. From (23), we see that P_T' and S_T' are proportional to $(a_4 - 2a_6 X_{ss})$ and $(a_3 - a_5)$, respectively, in addition to other common factors. Indeed, in the factorization scheme, since $(a_4 - 2a_6 X_{ss})$ and $(a_3 - a_5)$ have the same sign (all positive), the combination $(2P_T' + 4S_T')$ appearing in $B^{+(0)} \to \eta' K_2^{*+(0)}$ causes constructive interference, while the combination $(-P_T' + S_T')$ appearing in $B^{+(0)} \to \eta K_2^{*+(0)}$ causes destructive interference (see the appendix). Thus the predictions for these decay modes are consistent in both approaches. The modes $B^+ \to \pi^0 K_2^{*+}$ and $B^0 \to \pi^0 K_2^{*0}$ have almost the same branching ratios of $O(10^{-8})$ in the factorization scheme (also see the appendix). In flavor SU(3) symmetry, as shown in Table 2, the decay amplitudes for these modes have contributions from P_P' or T_P'. Thus, as discussed in the previous section, if P_P' or T_P' is not very suppressed compared to C_T', then there would be a sizable discrepancy in $\mathcal{B}(B^+ \to \pi^0 K_2^{*+}) \approx \mathcal{B}(B^0 \to \pi^0 K_2^{*0})$, and in principle this can be tested in experiment. The terms $-\left(T_P' + P_P' + (2/3)P_{\mathrm{EW},P}^{C\prime}\right)$ and $P_P' - (1/3)P_{\mathrm{EW},P}^{C\prime}$ can be determined by measuring the branching ratios for $B^0 \to \pi^- K_2^{*+}$ and $B^+ \to \pi^+ K_2^{*0}$, respectively. The CP asymmetries $\mathcal{A}_{CP}$ in $|\Delta S| = 1$ decays are shown in Table 6. The $\mathcal{A}_{CP}$'s in most modes are expected to be quite small. In $B^+(0) \to \eta K_2^{*+(0)}$, $\mathcal{A}_{CP}$ can be as large as 92%, but the corresponding branching ratio is as small as about $O(10^{-9})$.

5 Conclusion

We have analyzed exclusive charmless decays, $B \to PT$, in the schemes of both flavor SU(3) symmetry and generalized factorization. Using the flavor SU(3) symmetry, we have decomposed all the amplitudes for decays $B \to PT$ into linear combinations of the relevant SU(3) amplitudes. Based on the decomposition, we have shown that certain decay modes, such as $B^+ \to \pi^+ a_2^0$, $\pi^+ f_2$ and $B^0 \to \pi^+ a_2^-$ in $\Delta S = 0$ decays, and $B^{+(0)} \to \eta' K_2^{*+(0)}$ in $|\Delta S| = 1$ decays, are expected to have the largest decay rates, so these modes can be preferable to find in future experiments. Certain ways to test the validity of the factorization scheme have been presented by emphasizing the interplay between both approaches and carefully combining the predictions from both approaches. In order to bridge the flavor SU(3) approach and the factorization approach, we have explicitly presented a set of relations between a flavor SU(3) amplitude and the corresponding amplitude in factorization in $B \to PT$ decays. To calculate the branching ratios for $B \to PT$ decays, we have used the

Table 5. The branching ratios for $B \to PT$ decay modes with $|\Delta S| = 1$. The definitions for the columns are the same as those in Table 3

Decay mode	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$	$\mathcal{B}(10^{-8})$
$B^+ \to K^+ a_2^0$	4.31	5.77	3.81	5.08	3.34	4.43
$B^+ \to K^+ f_2$	4.69	6.27	4.14	5.52	3.63	4.82
$B^+ \to K^+ f_2'$	0.044	0.058	3.84	0.051	0.037	0.045
$B^+ \to K^0 a_2^+$	5.08	1.22	6.22	1.97	7.47	2.91
$B^+ \to \pi^0 K_2^{*+}$	1.13	1.55	1.09	1.05	1.19	0.75
$B^+ \to \eta K_2^{*+}$	0.10	0.23	0.035	0.22	0.077	0.31
$B^+ \to \eta' K_2^{*+}$	43.09	26.58	44.96	29.98	46.91	33.64
$B^0 \to K^+ a_2^-$	8.16	10.92	7.21	9.61	6.32	8.39
$B^0 \to K^0 a_2^0$	2.40	0.58	2.94	0.93	3.53	1.38
$B^0 \to K^0 f_2$	2.61	0.63	3.20	1.01	3.84	1.50
$B^0 \to K^0 f_2'$	0.024	0.006	0.030	0.009	0.036	0.014
$B^0 \to \pi^0 K_2^{*0}$	1.05	1.45	1.02	0.98	1.11	0.70
$B^0 \to \eta K_2^{*0}$	0.095	0.21	0.033	0.21	0.072	0.29
$B^0 \to \eta' K_2^{*0}$	40.14	24.76	41.88	27.93	43.70	31.34

full effective Hamiltonian including all the penguin operators which are essential to analyze the $|\Delta S| = 1$ processes and to calculate CP asymmetries. We have also used the non-relativistic quark model proposed by Isgur, Scora, Grinstein, and Wise to obtain the form factors describing $B \to T$ transitions. As shown in Tables 3 and 5, the branching ratios vary from $O(10^{-7})$ to $O(10^{-10})$.

Consistent with the prediction from the flavor SU(3) analysis, the decay modes such as $B^+ \to \pi^+ a_2^0$, $\pi^+ f_2$, $B^0 \to \pi^+ a_2^-$ and $B^{+(0)} \to \eta' K_2^{*+(0)}$ as well as $B^+ \to \eta' a_2^+$ have branching ratios of order of 10^{-7}. In particular, the branching ratio for the mode $B^0 \to \pi^+ a_2^-$ can be as large as almost $O(10^{-6})$. We have identified the decay modes where the CP asymmetries are expected to be large, such as $B \to \eta' a_2^+$, ηa_2^+, ηa_2^0, ηf_2 in $\Delta S = 0$ decays, and $B^+(0) \to \eta K_2^{*+(0)}$ in $|\Delta S| = 1$ decays. Due to possible uncertainties in the hadronic form factors of $B \to PT$ and non-factorizaton effects, the predicted branching ratios could be increased. Although experimentally challenging, the exclusive charmless decays, $B \to PT$, can probably be carried out in detail in hadronic B experiments such as BTeV and LHC-B, where more than 10^{10} B mesons will be produced per year, as well as at present asymmetric B factories of Belle and Babar.

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Appendix

In this appendix, we present expressions for all the decay amplitudes of $B \to PT$ modes shown in Tables 1 and 2

Table 6. The CP asymmetries for $B \to PT$ decay modes with $|\Delta S| = 1$. The definitions for the columns are the same as those in Table 3

Decay mode	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$	$\mathcal{A}_{CP}$
$B^+ \to K^+ a_2^0$	−0.11	0.022	−0.11	0.022	−0.11	0.022
$B^+ \to K^+ f_2$	−0.12	0.022	−0.11	0.022	−0.11	0.022
$B^+ \to K^+ f_2'$	−0.12	0.022	−0.11	0.022	−0.11	0.022
$B^+ \to K^0 a_2^+$	0	0	0	0	0	0
$B^+ \to \pi^0 K_2^{*+}$	0.006	0.004	−0.001	−0.001	−0.007	−0.010
$B^+ \to \eta K_2^{*+}$	0.65	0.39	−0.21	−0.043	−0.92	−0.31
$B^+ \to \eta' K_2^{*+}$	0.005	0.006	−0.001	−0.001	−0.005	−0.005
$B^0 \to K^+ a_2^-$	−0.12	0.022	−0.11	0.022	−0.11	0.022
$B^0 \to K^0 a_2^0$	0	0	0	0	0	0
$B^0 \to K^0 f_2$	0	0	0	0	0	0
$B^0 \to K^0 f_2'$	0	0	0	0	0	0
$B^0 \to \pi^0 K_2^{*0}$	0.006	0.004	−0.001	−0.001	−0.007	−0.010
$B^0 \to \eta K_2^{*0}$	0.65	0.39	−0.21	−0.043	−0.92	−0.31
$B^0 \to \eta' K_2^{*0}$	0.005	0.006	−0.001	−0.001	−0.005	−0.005

as calculated in the factorization scheme. Below we use $F^{B\to T}$ and $X_{qq'}$ defined in (24) and (25).

(1) $B \to PT$ ($\Delta S = 0$) decays.

$$\begin{aligned} &A(B^+ \to \pi^+ a_2^0) \\ &= \mathrm{i}\frac{G_{\mathrm{F}}}{2} f_\pi \epsilon^*_{\mu\nu} p_B^\mu p_B^\nu F^{B\to a_2^0}(m_\pi^2) \\ &\quad \times \{V_{ub}^* V_{ud} a_1 - V_{tb}^* V_{td}[a_4 + a_{10} - 2(a_6 + a_8) X_{du}]\}, \end{aligned} \tag{30}$$

$$\begin{aligned} &A(B^+ \to \pi^+ f_2) \\ &= \mathrm{i}\frac{G_{\mathrm{F}}}{2} \cos\phi_T f_\pi \epsilon^*_{\mu\nu} p_B^\mu p_B^\nu F^{B\to f_2}(m_\pi^2) \\ &\quad \times \{V_{ub}^* V_{ud} a_1 - V_{tb}^* V_{td}[a_4 + a_{10} - 2(a_6 + a_8) X_{du}]\}, \end{aligned} \tag{31}$$

$$A(B^+ \to \pi^+ f_2') \tag{32}$$
$$= \mathrm{i}\frac{G_F}{2}\sin\phi_T f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to f_2}(m^2_\pi) \times \{V^*_{ub}V_{ud}a_1 - V^*_{tb}V_{td}[a_4 + a_{10} - 2(a_6+a_8)X_{du}]\}\,,$$

$$A(B^+ \to \pi^0 a_2^+) \tag{33}$$
$$= \mathrm{i}\frac{G_F}{2} f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^+}(m^2_\pi) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[-a_4 + \frac{3}{2}a_7 - \frac{3}{2}a_9 + \frac{1}{2}a_{10} + 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^+ \to \eta a_2^+) \tag{34}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{\sqrt{3}} f_\eta \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^+}(m^2_\eta) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[a_3 + a_4 - a_5 + a_7 - a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^+ \to \eta' a_2^+) \tag{35}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{\sqrt{6}} f_{\eta'} \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^+}(m^2_{\eta'}) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[4a_3 + a_4 - 4a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^+ \to \bar{K}^0 K_2^{*+}) \tag{36}$$
$$= -\mathrm{i}\frac{G_F}{\sqrt{2}} f_K \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to K_2^{*+}}(m^2_K) V^*_{tb}V_{td} \times \left[a_4 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{ds}\right],$$

$$A(B^0 \to \pi^+ a_2^-) \tag{37}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}} f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^-}(m^2_\pi) \times \{V^*_{ub}V_{ud}a_1 - V^*_{tb}V_{td}[a_4 + a_{10} - 2(a_6+a_8)X_{du}]\}\,,$$

$$A(B^0 \to \pi^0 a_2^0) \tag{38}$$
$$= \mathrm{i}\frac{G_F}{2\sqrt{2}} f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^0}(m^2_\pi) \times \left\{V^*_{ub}V_{ud}(-a_2) - V^*_{tb}V_{td}\left[a_4 - \frac{3}{2}a_7 + \frac{3}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \pi^0 f_2) \tag{39}$$
$$= \mathrm{i}\frac{G_F}{2\sqrt{2}}\cos\phi_T f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to f_2}(m^2_\pi) \times \left\{V^*_{ub}V_{ud}(-a_2) - V^*_{tb}V_{td}\left[a_4 - \frac{3}{2}a_7 + \frac{3}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \pi^0 f_2') \tag{40}$$
$$= \mathrm{i}\frac{G_F}{2\sqrt{2}}\sin\phi_T f_\pi \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to f_2'}(m^2_\pi) \times \left\{V^*_{ub}V_{ud}(-a_2) - V^*_{tb}V_{td}\left[a_4 - \frac{3}{2}a_7 + \frac{3}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \eta a_2^0) \tag{41}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{\sqrt{6}} f_\eta \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^0}(m^2_\eta) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[a_3 + a_4 - a_5 + a_7 - a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \eta f_2) \tag{42}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{\sqrt{6}}\cos\phi_T f_\eta \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to f_2}(m^2_\eta) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[a_3 + a_4 - a_5 + a_7 - a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \eta f_2') \tag{43}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{\sqrt{6}}\sin\phi_T f_\eta \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to f_2'}(m^2_\eta) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[a_3 + a_4 - a_5 + a_7 - a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \eta' a_2^0) \tag{44}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{2\sqrt{3}} f_{\eta'} \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to a_2^0}(m^2_{\eta'}) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[4a_3 + a_4 - 4a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \eta' f_2) \tag{45}$$
$$= \mathrm{i}\frac{G_F}{\sqrt{2}}\frac{1}{2\sqrt{3}}\cos\phi_T f_{\eta'} \epsilon^*_{\mu\nu} p^\mu_B p^\nu_B F^{B\to f_2}(m^2_{\eta'}) \times \left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[4a_3 + a_4 - 4a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \eta' f_2') \tag{46}$$

$$= \mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{1}{2\sqrt{3}}\sin\phi_T f_{\eta'}\epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to f_2}(m^2_{\eta'}) \times\left\{V^*_{ub}V_{ud}a_2 - V^*_{tb}V_{td}\left[4a_3 + a_4 - 4a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{dd}\right]\right\},$$

$$A(B^0 \to \bar{K}^0 K_2^{*0}) \quad (47)$$
$$= -\mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}} f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*0}}(m^2_K)V^*_{tb}V_{td} \times\left[a_4 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{ds}\right],$$

(2) $B \to PT$ ($|\Delta S| = 1$) decays.

$$A(B^+ \to K^+ a_2^0) \quad (48)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2} f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to a_2^0}(m^2_K), \times\{V^*_{ub}V_{us}a_1 - V^*_{tb}V_{ts}[a_4 + a_{10} - 2(a_6 + a_8)X_{su}]\}$$

$$A(B^+ \to K^+ f_2) \quad (49)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2}\cos\phi_T f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to f_2}(m^2_K) \times\{V^*_{ub}V_{us}a_1 - V^*_{tb}V_{ts}[a_4 + a_{10} - 2(a_6 + a_8)X_{su}]\},$$

$$A(B^+ \to K^+ f_2') \quad (50)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2}\sin\phi_T f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to f_2'}(m^2_K) \times\{V^*_{ub}V_{us}a_1 - V^*_{tb}V_{ts}[a_4 + a_{10} - 2(a_6 + a_8)X_{su}]\},$$

$$A(B^+ \to \bar{K}^0 a_2^+) \quad (51)$$
$$= -\mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}} f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to a_2^+}(m^2_K)V^*_{tb}V_{ts} \times\left[a_4 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{sd}\right],$$

$$A(B^+ \to \pi^0 K_2^{*+}) \quad (52)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2} f_\pi \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*+}}(m^2_\pi) \times\left[V^*_{ub}V_{us}a_2 - V^*_{tb}V_{ts}\left(\frac{3}{2}a_7 - \frac{3}{2}a_9\right)\right],$$

$$A(B^+ \to \eta K_2^{*+}) \quad (53)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{1}{\sqrt{3}} f_\eta \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*+}}(m^2_\eta) \times\left\{V^*_{ub}V_{us}a_2 - V^*_{tb}V_{ts}\left[a_3 - a_4 - a_5 + a_7 - a_9 + \frac{1}{2}a_{10} + 2\left(a_6 - \frac{1}{2}a_8\right)X_{ss}\right]\right\},$$

$$A(B^+ \to \eta' K_2^{*+}) \quad (54)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{1}{\sqrt{6}} f_{\eta'} \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*+}}(m^2_{\eta'}) \times\left\{V^*_{ub}V_{us}a_2 - V^*_{tb}V_{ts}\left[4a_3 + 2a_4 - 4a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 - a_{10} - 4\left(a_6 - \frac{1}{2}a_8\right)X_{ss}\right]\right\},$$

$$A(B^0 \to K^+ a_2^-) \quad (55)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}} f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to a_2^-}(m^2_K) \times\{V^*_{ub}V_{us}a_1 - V^*_{tb}V_{ts}[a_4 + a_{10} - 2(a_6 + a_8)X_{su}]\},$$

$$A(B^0 \to K^0 a_2^0) \quad (56)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2} f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to a_2^0}(m^2_K)V^*_{tb}V_{ts} \times\left[a_4 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{sd}\right],$$

$$A(B^0 \to K^0 f_2) \quad (57)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2}\cos\phi_T f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to f_2}(m^2_K)V^*_{tb}V_{ts} \times\left[a_4 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{sd}\right],$$

$$A(B^0 \to K^0 f_2') \quad (58)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2}\sin\phi_T f_K \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to f_2'}(m^2_K)V^*_{tb}V_{ts} \times\left[a_4 - \frac{1}{2}a_{10} - 2\left(a_6 - \frac{1}{2}a_8\right)X_{sd}\right],$$

$$A(B^0 \to \pi^0 K_2^{*0}) \quad (59)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{2} f_\pi \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*0}}(m^2_\pi) \times\left[V^*_{ub}V_{us}a_2 - V^*_{tb}V_{ts}\left(\frac{3}{2}a_7 - \frac{3}{2}a_9\right)\right],$$

$$A(B^0 \to \eta K_2^{*0}) \quad (60)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{1}{\sqrt{3}} f_\eta \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*0}}(m^2_\eta) \times\left\{V^*_{ub}V_{us}a_2 - V^*_{tb}V_{ts}\left[a_3 - a_4 - a_5 + a_7 - a_9 + \frac{1}{2}a_{10} + 2\left(a_6 - \frac{1}{2}a_8\right)X_{ss}\right]\right\},$$

$$A(B^0 \to \eta' K_2^{*0}) \quad (61)$$
$$= \mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{1}{\sqrt{6}} f_{\eta'} \epsilon^*_{\mu\nu}p^\mu_B p^\nu_B F^{B\to K_2^{*0}}(m^2_{\eta'}) \times\left\{V^*_{ub}V_{us}a_2 - V^*_{tb}V_{ts}\left[4a_3 + 2a_4 - 4a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 - a_{10} - 4\left(a_6 - \frac{1}{2}a_8\right)X_{ss}\right]\right\}.$$

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